

# 1.6 Perform Operations with Complex Numbers

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$x^2 = -1$  has **NO REAL** number solutions because the square of any real number  $x$  is never a negative number.

To overcome this problem, mathematicians created an expanded system of numbers using the imaginary unit  $i$ , defined as  $i = \sqrt{-1}$

## The square root of a negative number

Property	Example
1. If $r$ is a positive real number, then $\sqrt{-r} = i\sqrt{r}$	$\sqrt{-3} = i\sqrt{3}$
2. By property (1), it follows that $(i\sqrt{r})^2 = -r$	$(i\sqrt{3})^2 = i \cdot 3 = -3$

## Solve a Quadratic Equation

$2x^2 + 11 = -37$	Subtract 11
$2x^2 = -48$	Divide by 2
$x^2 = -24$	Square Root
$x = \pm\sqrt{-24}$	Write in term of $i$ (pull out negative under the radical)
$x = \pm i\sqrt{24}$	Simplify the radical
$x = \pm 2i\sqrt{6}$	

**Complex Number:**  $a + bi$   $a$  is the real part,  $bi$  is the imaginary part

### Multiply Complex Numbers

$(9 - 2i)(-4 + 7i)$	FOIL
$-36 + 63i + 8i - 14i^2$	Simplify, Replace $i^2$ with $-1$
$-36 + 71i - 14(-1)$	Simplify
$-36 + 71i + 14$	Combine like terms
$-22 + 71i$	

### Divide Complex Numbers

$\frac{7 + 5i}{1 - 4i}$	Multiply numerator and denominator by the complex conjugate
$\frac{7 + 5i}{1 - 4i} \cdot \frac{1 + 4i}{1 + 4i}$	FOIL FL
$\frac{-13 + 33i}{17}$	

Plot Complex numbers: real-x value imaginary-y value

3-2i	(3,-2i)
-2+4i	(-2,4)
5i	(0,5)
-7	(-7,0)

Absolute Value:  $=\sqrt{a^2 + b^2}$

3-4i

$$\sqrt{3^2 + (-4)^2}$$

$$\sqrt{9 + 16}$$

$$\sqrt{25}$$

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